WEEK $5\frac{1}{2}$

We saw that the cross-section for scattering of an electron from a "fixed" nucleus (Rutherford scattering) was given by

$$\frac{d\sigma}{d\Omega_f} = \frac{4Z^2\alpha^2}{q^4} m^2 \, |u_f^+ u_i|^2$$

$$S = \frac{1}{2} \sum_{\lambda_1} \sum_{\lambda_2} |u^+(p_f \lambda_f) \, u(p_i \lambda_i)|^2$$

$$S = \frac{1}{2} \, T_r \left\{ \Lambda^+(p_f)\gamma^0 \, \Lambda^+(p_i)\gamma^0 \right\}$$

$$= \frac{1}{8m^2} \, T_r \left\{ (\not{p}_f + m)\gamma^0 \, (\not{p}_i + m)\gamma^0 \right\}$$

$$= \frac{1}{8m^2} \, T_r \left\{ m^2 \mathbb{1} + \not{p}_f \gamma^0 m \mathbb{1} + m\gamma^0 \not{p}_i + \not{p}_f \gamma^0 \not{p}_i \gamma^0 \right\}$$

$$= \frac{1}{8m^2} \, T_r \left\{ m^2 + \not{p}_f \gamma^0 \not{p}_i \gamma^0 \right\}.$$

now $\not{p}\gamma^0 = (\gamma^0 p^0 - \underline{\gamma} \cdot \underline{p})\gamma^0 = \gamma^0(\gamma^0 p^0 + \underline{\gamma} \cdot \underline{p})$

so let $\tilde{p} = (p^0, -\underline{p})$

$$\boxed{\therefore \not{p}\gamma^0 = \gamma^0 \tilde{\not{p}}}$$

$$\therefore \quad S = \frac{1}{8m^2} \operatorname{Tr}\left(m^2 \quad \not{p}_f \tilde{\not{p}}_i \right)$$

$$= \frac{1}{8m^2} \left\{ 4m^2 + 4 p_f . \tilde{p}_i \right\}$$

$$= \left(\frac{E}{m}\right)^2 \left(1 - v^2 \sin^2 \theta/_2 \right).$$

So what about the current interpretation on the spin $1/2$ Feynman graphs.

Going back to

$$a_f^{(1)} = i \int dt \int d^3x \, \psi_f^+ (e^- \, p_f, \lambda_f) \, V^{(1)} \psi_i (e^-, p_i, \lambda_i)$$

where $V^{(1)} = \left(e \, \underline{\alpha} . \underline{A} - e A^0 \right)$

$$a_f^{(1)} = -i \int \psi_f^+ (x) \, \left(e \, \underline{\alpha} . \underline{A}(x) - e A^0(x) \right) \psi_i (x) \, d^4 x$$

but using $u^+ = \bar{u} \gamma^0 \qquad \underline{\gamma} = \gamma^0 \, \underline{\alpha}.$

We get

$$a_f^{(1)} = -\int \bar{\Psi}_f(x) \{ -e\gamma^\mu A_\mu(x) \} \Psi_i(x)\, d^4x .$$

If $A^\mu(x)$ is independent of t

$$a_f^{(1)} = -i2\pi\,\delta(E_f - E_i)\, V_{fi}$$

$$V_{fi} = -e\int \bar{\Psi}_f(x)\,\gamma^\mu\,\Psi_i(x)\, A_\mu(x)\, d^3x .$$

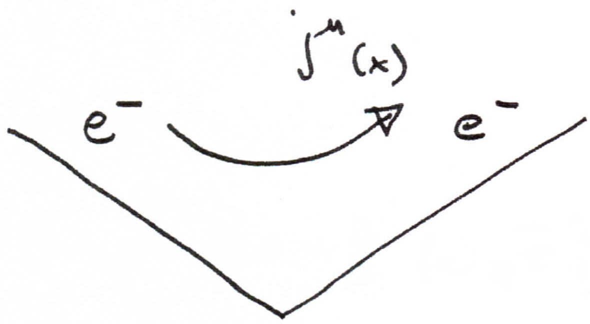
Before we had
(K.G)

$$V_{fi} = -e\int j^\mu_{current}\, A_\mu(x)\, d^3x .$$

Thus for Dirac spin-½ particles we can write

$$\boxed{j^\mu_{current} = -e\,\bar{\Psi}_f(x)\,\gamma^\mu\,\Psi_i(x)}$$

thus



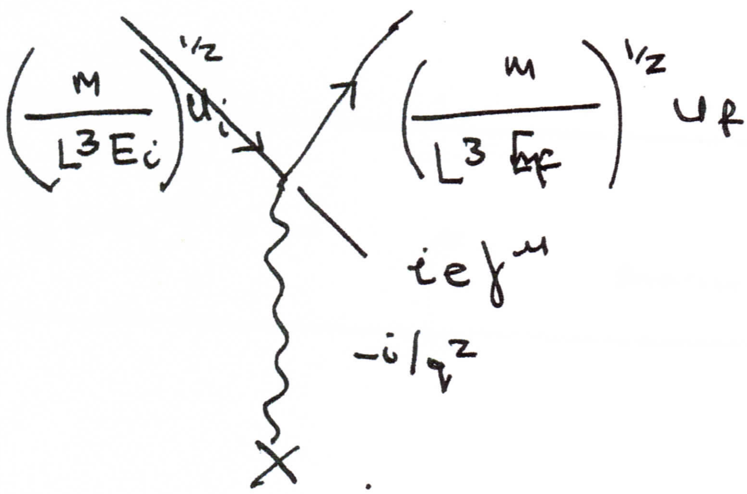
$$j^{\mu}(x) = -e\,\bar{\psi}_f \gamma^{\mu} \psi .$$

$$= -e \left(\frac{1}{L^3}\right)^{1/2} \left(\frac{m}{E_f}\right)^{1/2} e^{i p_f^{\mu} x_{\mu}}$$

$$\bar{u}_f \gamma^{\mu} u_i$$

$$\left(\frac{1}{L^3}\right)^{1/2} \left(\frac{m}{E_i}\right)^{1/2} e^{-i p_i^{\mu} x_{\mu}} .$$

This gives us the Feynman diagram.



It can be shown that

This is called magnetic part.

$$\bar{u}(p_f)\gamma^{\mu} u(p_i) = \bar{u}(p_f)\left\{ \frac{(p_i + p_f)^{\mu}}{2m} + i\,\frac{\sigma^{\mu\nu} q_{\nu}}{2m} \right\} u(p_i).$$

This is spin-0 current!

where $\sigma^{\mu\nu} = (i/2)\left(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}\right)$

Prove if $(\not{p}_i - m)\, u(p_i) = 0$.

prove $\bar{u}(p_f)\,(\not{p}_f - m) = 0$.

or really. $(\not{p} - m)\, u(p) = 0$.

$\bar{u}\,(\not{p} - m) = 0$.

And prove

$$\bar{u}(p_f)\left\{ \frac{(p_i + p_f)^{\mu}}{2m} + \frac{i\,\sigma^{\mu\nu} q_{\nu}}{2m} \right\} u(p_i)$$

where $\sigma^{\mu\nu} = (i/2)\left(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}\right)$.