$\propto |a_{if}|$ ... $_{if}$ ...

$$a_{if} = \iiint_{-\infty}^{\infty} \psi_f^* \, V(x) \, \psi_i \, dx^3 \int_{-\infty}^{\infty} e^{-iE_f t} \, e^{i\omega t} \, e^{iE_i t} \, d$$

$$\text{if} \quad \psi_E(x,t) = \psi(x) \, e^{-iEt} .$$

$E = E_f - \ldots$!

Now if we have an integral of the form.

$$\int_{-\infty}^{\infty} e^{-iEt} . dt = \lim_{T\to\infty} \int_{-T}^{T} e^{-iEt} . dt$$

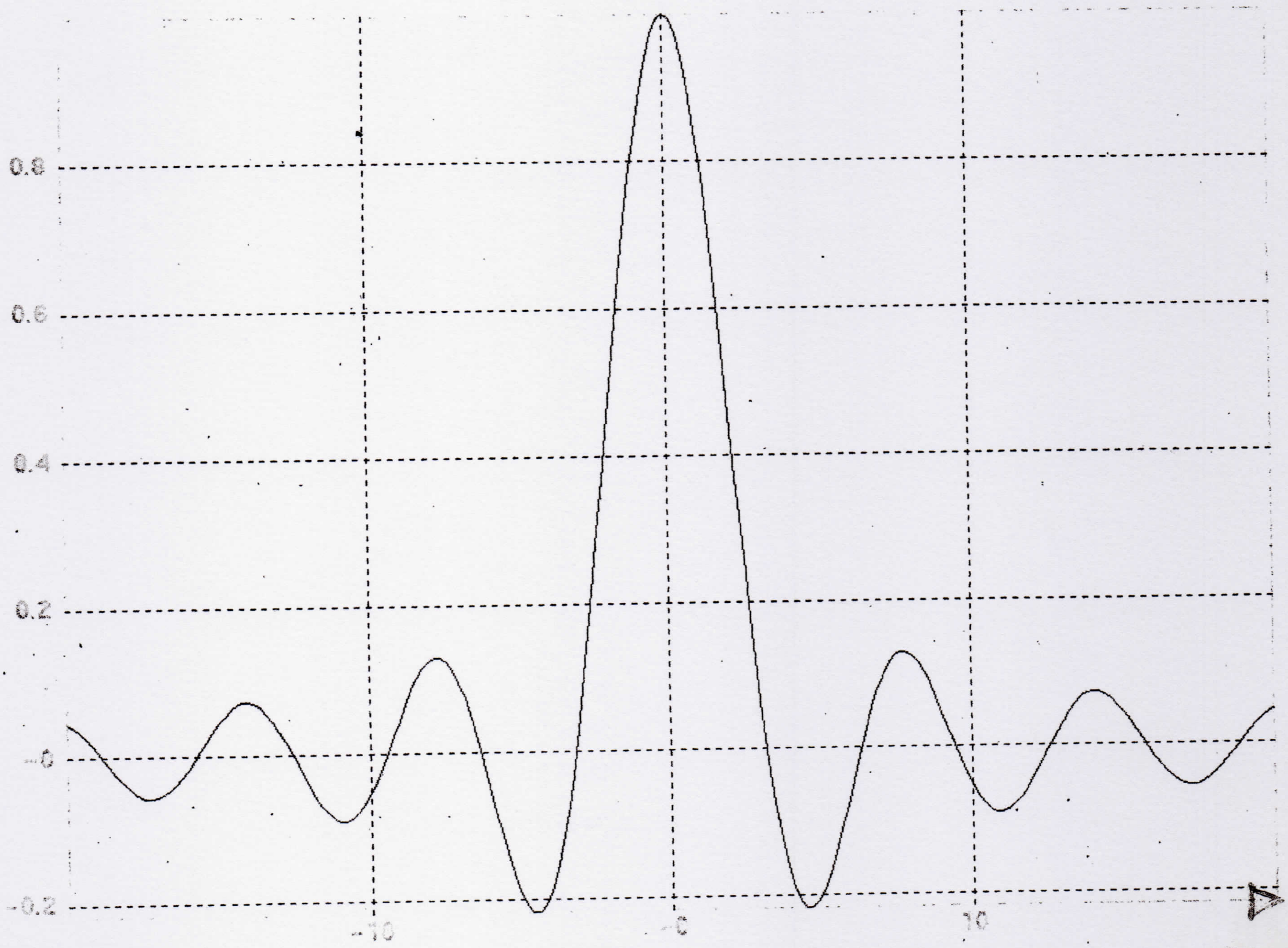
$$= \lim_{T\to\infty} \left[-\frac{1}{iE} e^{-iEt} \right]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{-1}{iE} \; e^{-iET} - e^{iET}$$

$$= \lim_{T\to\infty} 2i \frac{\sin ET}{E}$$

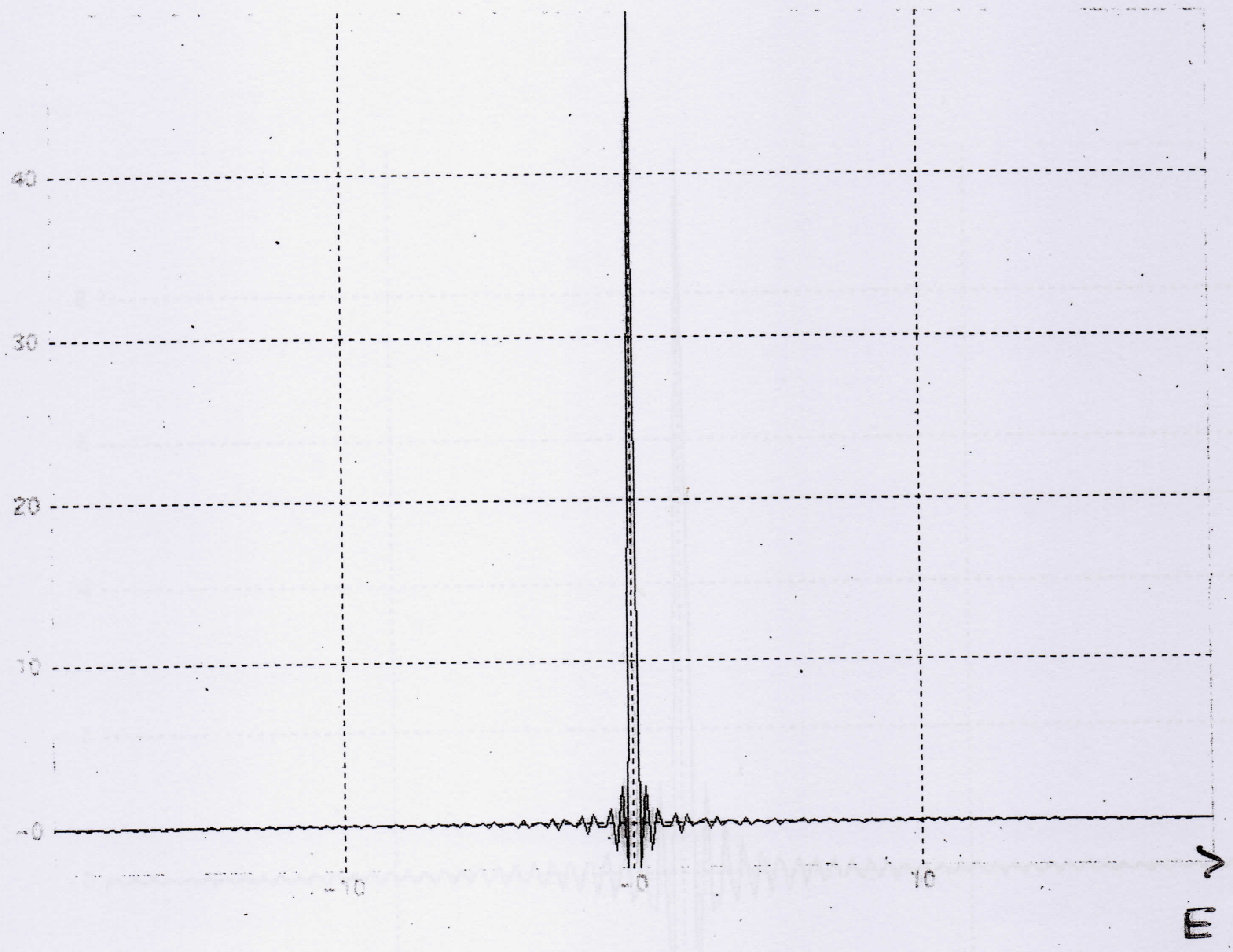
What does this function look like:

T = 1



4a.

T = 50



$g(z)$

$$\oint \frac{f(z)}{(z-z_0)}\,dz = 2\pi i\, f(z_0) \text{ if pole inside contour.}$$

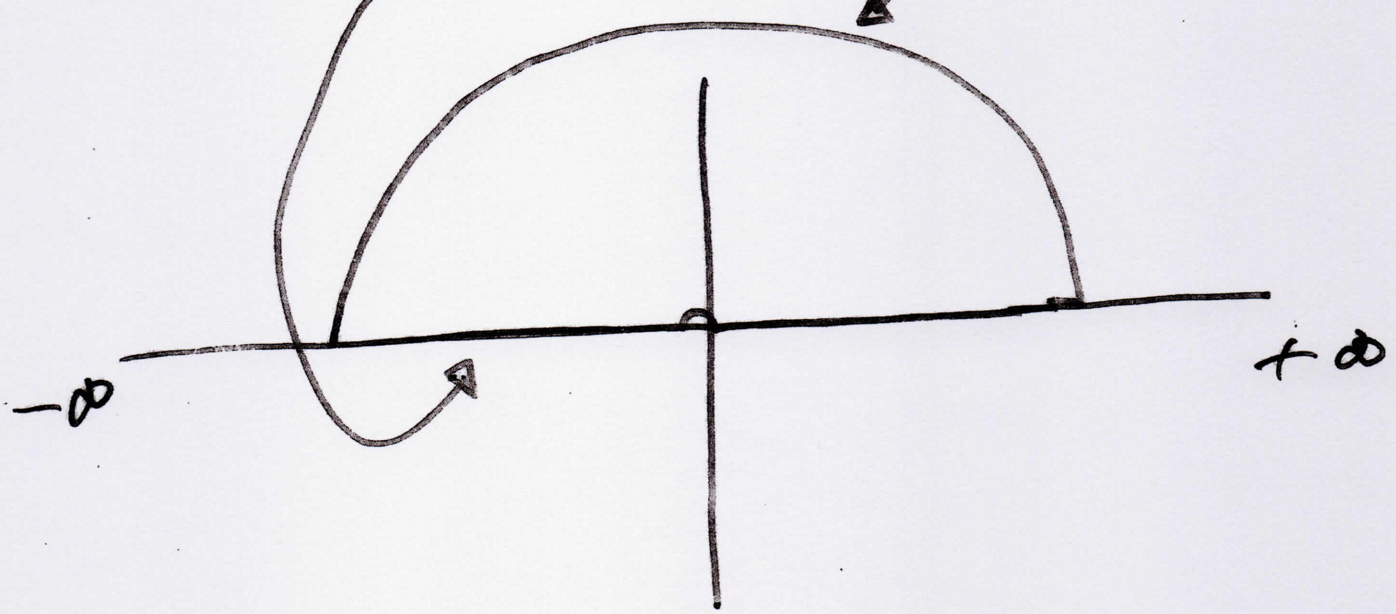
$(\pi i\, f(z_0))$ if pole on contour.

Since $z \to \infty$

But

$$\lim_{t\to\infty} 2i \int_{-\infty}^{\infty} \frac{e^{iz}}{z}\,dz + \int_{C} \frac{e^{iz}}{z}\,dz = \oint \frac{e^{iz}}{z}\,dz.$$

$= 0$.



$$\therefore \lim_{t\to\infty} 2i \int_{-\infty}^{\infty} \frac{e^{iz}}{z}\,dz = \lim_{t\to\infty} 2\pi i e^{i0}$$

$$= 2\pi.$$

Finally what other properties does this (7)

Function have.

$$\lim_{t \to \infty} 2i \frac{\sin Et}{E}$$

(1) as $t \to \infty$ it is only nonzero

at $E = 0$

(2) The integral is a constant.

$$\lim_{t \to \infty} \int_{-\infty}^{\infty} 2i \frac{\sin Et}{E} \, dt$$

$$= \underset{t \to \infty}{\text{Imaginary} \lim} \int_{-\infty}^{\infty} 2i \frac{e^{iEt}}{E} \, dt$$

change variable to $Z = Et$

$$= \underset{t \to \infty}{\text{Imag} \lim} \int_{-\infty}^{\infty} 2i \frac{e^{iZ}}{Z} \, dZ$$

Note that 1 $\frac{\sin TE}{E} \sim \frac{TE}{E} = T$

for small TE $\therefore$ height of peak $= T$.

2 First minimum is at

$\sin TE = 0$ $\therefore$ $TE = \pm \pi$.

Thus $\Delta E = \frac{2\pi}{T}$

But T is really ΔT the time for which this new state has existed.

$\therefore$ $\boxed{\Delta T \Delta E = 2\pi}$

Heisenberg's Uncertainty Principle

Thus

$$\lim_{t \to \infty} \frac{1}{2\pi} 2i \frac{\sin Et}{E} = \delta(E)$$

where $\int \delta(E)\, dE = 1$

and $\int f(E)\, \delta(E)\, dE = f(0)$

$$\boxed{\text{The DIRAC } \delta\text{- function}}$$

In this case

$$E = -E_f + \omega + E_i = 0 .$$

$$\therefore E_f = E_i + \omega$$

$\therefore$ conservation of energy.